Z Tests

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Overview of Z Tests

We will begin with the Z-test.

The Z-test for proportions is a statistical method used to compare observed proportions, such as the proportion of successes in two groups, to see if there is a significant difference between them. This test is particularly useful when dealing with large sample sizes.

Depending on the research question and the type of data being analyzed, different types of Z-tests are frequently employed in statistics. Variations of the Z-test can be used to compare sample means or sample proportions. In this chapter, we will confine our attention to the Z-tests used to analyze **categorical** data and **sample proportions**.

The following are three prevalent types of Z-tests for Categorical data:

1. **One-sample Z-test:** This test determines whether a sample proportion significantly deviates from a known or hypothesized population mean or proportion.
2. **Two-sample Z-test:** This test is used to determine whether there is a statistically significant difference between the proportions of two independent groups.
3. **Paired Z-test for proportions:** This test determines whether there is a statistically significant difference in proportions between two related groups, such as before and after a treatment or intervention. [1]

Business Applications of Z Tests for Categorical Data

Marketing

In marketing research and analysis, Z-tests for categorical data are useful tools for testing hypotheses and determining the significance of relationships between categorical variables. Here are some possible marketing applications of Z-tests for categorical data:

1. **Brand loyalty:** A marketer may wish to determine whether customers who purchase a particular brand are more loyal to the brand based on repeat purchase, than those who purchase a competing brand. To compare the proportion of loyal customers between the two brands, a two-sample Z-test for proportions could be utilized.

2. **Customer satisfaction:** A marketer may wish to determine whether a new product or service is more satisfying to customers than its predecessor. Before and after the introduction of the new product or service, the proportion of satisfied customers could be compared.

3. **Market segmentation:** A marketer may use market segmentation to determine whether there are significant differences in customer demographics or behavior across market segments. A test could be utilized to determine if there is a significant relationship between the demographic variables and market segments.

4. **Advertising effectiveness:** A marketer may wish to determine if one advertising campaign is more effective than another at reaching the target audience. A two-sample Z-test for proportions could be used to compare the proportion of viewers who remember the message or take action following exposure to each campaign.

5. **Product preference:** A marketer may wish to determine whether a particular product feature or characteristic is more preferred by customers than another. A one-sample Z-test for proportions could be utilized to determine if the proportion of customers who prefer the feature significantly deviates from the null hypothesis. [2]

Finance

Z tests can be helpful in studying different aspects of Finance such as default rates, investment decisions, stock market analysis, risk analysis.

1. **Credit risk assessment:** A financial institution assessing credit risk may wish to determine whether the proportion of delinquent loans varies significantly across risk categories. A two-sample Z-test for proportions could be utilized to compare the proportion of delinquent loans across various risk categories.

2. **Investment portfolio analysis:** An investor may wish to determine whether the proportion of winning trades varies significantly between different types of investments, including stocks and bonds. Using a two-sample Z-test for proportions, it is possible to compare the proportion of profitable trades for the various types of investments.

3. **Customer behavior analysis:** A financial institution may conduct an analysis of customer behavior to determine whether the proportion of customers who use online banking varies significantly by age group. A Chi-square test could be utilized to determine if there is a significant relationship between the demographic variable (age group) and the proportion of customers who use online banking.

4. **Market analysis:** An investment bank may wish to determine if there is a significant correlation between the market type (bullish or bearish) and the proportion of investors who sell or purchase stocks. A Chi-square test could be used to determine if there is a significant relationship between the type of market and the proportion of investors who sell or purchase stocks.

5. **Fraud detection:** A financial institution may wish to determine whether the proportion of fraudulent transactions varies significantly between different types of transactions, such as ATM withdrawals, online purchases, and wire transfers, for the purpose of detecting fraud. A one-sample Z-test for proportions could be used to determine if the proportion of fraudulent transactions in a specific type of transaction significantly deviates from a null hypothesis. [3]

Organizational Behavior:

In organizational behavior research, Z-tests for categorical data can be useful tools for analyzing data and making informed decisions. Here are some potential organizational behavior applications of Z-tests for categorical data:

1. **Employee satisfaction:** An organization may wish to determine whether the proportion of satisfied employees varies significantly across departments. Using a two-sample Z-test for proportions, it is possible to compare the percentage of satisfied employees across departments.

2. **Diversity and inclusion:** An organization may wish to determine whether the proportion of employees from diverse backgrounds varies significantly across management levels. A Chi-square test could be used to determine whether there is a significant relationship between the demographic variable (race, gender, etc.) and the management level.

3. **Employee engagement:** An organization may wish to determine whether the proportion of engaged employees differs significantly across teams or work units. Using a two-sample Z-test for proportions, it is possible to compare the proportion of engaged employees among various teams or work units.

4. **Training effectiveness:** An organization may wish to determine whether the percentage of employees who pass a certification exam differs significantly between those who have completed a training program and those who have not. The proportion of employees who pass the certification exam could be compared between the two groups using a two-sample Z-test for proportions.

5. **Employee turnover:** An organization may wish to determine whether the proportion of employees who voluntarily leave the organization varies significantly across job functions or departments. A Chi-square test could be used to determine whether there is a significant relationship between the job function or department and the percentage of employees who leave the organization voluntarily. [4]

One-sample Z-test for Proportions

1. This type of Z-test is used to examine a population proportion hypothesis.
2. For instance, a business may wish to determine whether the proportion of customers who are satisfied with their product exceeds 0.5.
3. The company could collect a sample of customers, determine the proportion of satisfied customers in the sample, and use a one-sample Z-test to determine if the proportion is significantly different from 0.5.

One-sample Z-test for Proportions - Concepts

1. The general steps to perform a one-sample Z-test are:

    1. **State the null hypothesis and alternative hypothesis:** Typically, the null hypothesis states that the sample proportion matches the hypothesized population proportion, whereas the alternative hypothesis states that the sample proportion differs from the hypothesized population proportion.
    2. **Determine the level of significance ( ):** This is the maximum probability of rejecting the null hypothesis when it is true. Typical values for are 0.05 or 0.01.
    3. **Collect sample data:** Obtain a random sample from the population and calculate the sample proportion (p).
    4. **Calculate the test statistic:** The test statistic is calculated as follows: $z = (p_0 - p)/\sqrt{(p * (1 - p)/n)}$, where p is the population proportion assumed under the null hypothesis, and n is the sample size.
    5. **Determine the critical value:** The critical value is the value of z that corresponds to the chosen level of significance and degrees of freedom ($df = n - 1$).
    6. **Compare the test statistic to the critical value:** If the absolute value of the test statistic is greater than the critical value, we reject the null hypothesis. Otherwise, we fail to reject the null hypothesis.
    7. **Interpret the results:** If the null hypothesis is rejected, one can conclude that the proportion of the sample is significantly different from that of the population. If the null hypothesis is not rejected, it can be concluded that the sample proportion is not significantly different from the population proportion.
    8. **Determine the p-value** using a Z-table or calculator, and compare the p-value to the level of significance (alpha) in order to reach a conclusion.
    9. **Draw a conclusion.** [5]

Numerical Example for One-sample Z-test for Proportions (Car Ownership)

1. **Research Objective**:
   A researcher wants to determine if the proportion of adults in a population who own a car significantly deviates from 70%. A random sample of 200 adults shows that 150 of them own a car.

2. **Hypotheses**:

   - Null Hypothesis (`H0`): `p = 0.7`
   - Alternative Hypothesis (`Ha`): `p   0.7`
   - Here, `p` is the population proportion.

3. **Significance Level**:
   The significance level is set at 0.05.

4. **Critical Value**:
   The critical value for a two-tailed test at the 0.05 significance level is ±1.96.

5. **Test Statistic Formula**:
   The test statistic (Z) is calculated using the formula:

$$Z = \frac{p_0 - p}{\sqrt{p \cdot (1 - p)/n}}$$

   - `p` is the hypothesized population proportion.
   - `p_0` is the sample proportion.
   - `n` is the sample size.

6. **Calculation**:

   - Sample proportion (`p_0`): `0.75` (150 out of 200)
   - Hypothesized population proportion (`p`): `0.7`
   - Sample size (`n`): `200`
   - The Z-score is calculated as approximately `1.54`.

Verify by plugging in the values:

$$Z = \frac{0.75 - 0.7}{\sqrt{0.7 \cdot (1 - 0.7)/200}}$$

The calculated Z-score is approximately `1.54`.

7. **Decision Rule**:

   - Compare the absolute value of the test statistic with the critical value. If it's greater than 1.96, reject the null hypothesis.

8. **Conclusion**:

   - Since the absolute value of the test statistic (1.54) is less than the critical value (1.96), we **fail to reject the null** hypothesis. There is not enough statistical evidence to conclude that the proportion of adults who own a car in the population is significantly different from 70%. [6]

Running One-sample Z-test for Proportions in R

1. Consider the `mtcars` data set. Suppose we want to test whether the proportion of cars with a **6-cylinder engine** (`cyl=6`) is significantly different from a hypothesized value of **50%**.

- **Null hypothesis**: The proportion of 6-cylinder engine cars in the `mtcars` dataset is 0.5 `(p = 0.5)`.
- **Alternative hypothesis:** The proportion of 6-cylinder engine cars in the `mtcars` dataset is not equal to 0.5 `(p   0.5)`.

```r
# Load the mtcars dataset
data(mtcars)

# Create a binary variable indicating whether a car has a 6-cylinder engine or not
mtcars$is_cyl6 <- ifelse(mtcars$cyl == 6, 1, 0)

# Count the number of 6 cylinder cars
n6 <- sum(mtcars$is_cyl6)

# Count the total number of cars
n <- length(mtcars$is_cyl6)
  # alternate code
  # n <- nrow(mtcars)

# Calculate the sample proportion of cars with a 6-cylinder engine
prop_cyl6 <- mean(mtcars$is_cyl6)

# Conduct a one-sample Z-test for proportions using the prop.test function
prop.test(n6,
          n,
          p = 0.5)
```

```
  1-sample proportions test with continuity correction
```

```
data:  n6 out of n, null probability 0.5
X-squared = 9.0312, df = 1, p-value = 0.002654
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.09944097 0.40441815
sample estimates:
      p
0.21875
```

2. The sample proportion is **0.21875**, indicating that only **21.875%** of the cars in the dataset are equipped with a six-cylinder engine.
3. Using the `prop.test` function, we can now perform a one-sample Z-test on proportions:
4. The `prop.test` function requires three arguments:

   the number of successes (in this case, the number of cars with a 6-cylinder engine),

   the total number of trials (in this case, the total number of cars), and

   the population proportion hypothesis (0.5 in this case)
5. The output reveals that the test statistic is X-squared = 9.0312 with a p-value of 0.002654 and one degree of freedom.
6. Since the p-value is less than the significance level (0.05), we reject the null hypothesis and conclude that the proportion of cars in the population with a 6-cylinder engine is significantly different from 50%.
7. The sample proportion estimate is 0.21875, with a 95% confidence interval encompassing this value (0.099, 0.404). [7]

Independent Samples Z-Test for Proportions (2)

1. **Introduction**: The independent samples Z-test for proportions is used to determine if there's a significant difference between two sample proportions drawn from independent populations.
2. **Example Scenario**: A company might want to know if the proportion of male customers who purchase their product is significantly different from the proportion of female customers.
3. **Application**: The company collects separate samples of male and female customers, calculates the purchase proportions, and uses an independent samples Z-test to assess the significance of any difference. [8]

Independent Samples Z-Test for Proportions - Concepts

Conducting the Test:

1. **Hypotheses**:

   - Null Hypothesis (H0): The two population proportions are equal.
   - Alternative Hypothesis (Ha): The two population proportions are not equal.

2. **Significance Level**: Typically set at 0.05, it's the acceptable risk level for incorrectly rejecting the null hypothesis.

3. **Test Statistic**: The test statistic is calculated using the formula:

$$Z = \frac{p1 - p2}{\sqrt{p \cdot (1 - p) \cdot (\frac{1}{n1} + \frac{1}{n2})}}$$

   where (p1) and (p2) are sample proportions, (n1) and (n2) are sample sizes, and (p) is the pooled sample proportion:

$$p = \frac{x1 + x2}{n1 + n2}$$

   (x1) and (x2) are the number of successes in each sample.

4. **Decision Rule**: The null hypothesis is rejected if the absolute value of (Z) is greater than the critical value (e.g., 1.96 for a two-tailed test at 0.05 significance level).

5. **Conclusion**: Based on the test results, a conclusion is drawn about the difference in population proportions. [8]

Numerical Example with Simulated Data for Independent Samples Z-Test

Scenario:

1. A researcher studies the purchasing behavior of men and women for a specific product. A sample of 250 men and 350 women is taken, with results showing that 125 men and 210 women purchased the product.

2. **Hypotheses**:

   - Null Hypothesis (H0): p1 = p2
   - Alternative Hypothesis (Ha): p1   p2

   where (p1) is the proportion of men, and (p2) is the proportion of women purchasing the product.

3. **Significance Level**: Set at 0.05.
4. **Critical Value**: For a two-tailed test at the 0.05 significance level, the critical value is approximately 1.96.
5. **Test Statistic Calculation**:

   The test statistic Z is calculated using the formula:

$$Z = \frac{p1 - p2}{\sqrt{p \cdot (1 - p) \cdot \left(\frac{1}{n1} + \frac{1}{n2}\right)}}$$

   where (p) is the pooled proportion, computed as:

$$p = \frac{x1 + x2}{n1 + n2}$$

6. **Using the Data**:

   For men:

$$p1 = \frac{125}{250} = 0.5$$

   For women:

$$p2 = \frac{210}{350} = 0.6$$

   Pooled proportion:

$$p = \frac{125 + 210}{250 + 350} = 0.55$$

7. **Test Statistic**:

   Calculation of the Z value:

$$Z = \frac{0.5 - 0.6}{\sqrt{0.55 \cdot (1 - 0.55) \cdot \left(\frac{1}{250} + \frac{1}{350}\right)}}$$

The calculated Z value is approximately -2.43. This value is negative because the proportion of women who purchased the product (p2 = 0.6) is higher than that of men (p1 = 0.5). However, since we are conducting a two-tailed test, the direction of the difference is not our primary concern. We are interested in whether the absolute value of the Z score is greater than the critical value.

8. **Conclusion**:

   Since (|Z| = 2.43) is greater than the critical value of 1.96, we reject the null hypothesis. This indicates that there is a significant difference in the purchasing behavior between men and women for this product.

9. **Interpretation**:

   With a 95% confidence level, we conclude that the proportions of men and women purchasing this product are significantly different. Specifically, the data suggests that women are more likely to purchase this product than men.

This example demonstrates a scenario where the Z-test reveals a significant difference in purchasing behaviors between the two groups, aligned with your request for a scenario where the proportions are different and the Z-value is positive. Note that even though the calculated Z-value is negative, its absolute value is used for the conclusion.

Running Independent Samples Z-Test in R (1)

Demonstration using simulated data:

To perform the test described in the scenario using R, we can use the prop.test function. This function is designed to handle comparisons of proportions. Here's how we can set it up with the data from the above scenario:

```r
x <- c(125, 210) # number of men and women who purchased the product
n <- c(250, 350) # total number of men and women

# Perform the two-proportion z-test
test_result <- prop.test(x,
                         n,
                         alternative = "two.sided",
                         correct = FALSE)
# Output the results
print(test_result)
```

```
	2-sample test for equality of proportions without continuity correction

data:  x out of n
X-squared = 5.9138, df = 1, p-value = 0.01502
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.18047113 -0.01952887
```

```
sample estimates:
prop 1 prop 2
   0.5    0.6
```

Notes about the R code

- `x` contains the number of successes (i.e., purchases) for each group (men first, then women).
- `n` contains the total number of observations for each group.
- `prop.test` is called with `correct = FALSE` to perform a standard two-proportion Z-test without continuity correction, which is appropriate for large sample sizes.
- we can specify alternative = `two.sided` in the `prop.test` function to explicitly indicate that we are performing a two-sided test. However, it's worth noting that the default setting for the alternative parameter in `prop.test` is already `two.sided`
- The function will return various information, including the Z statistic and the p-value.

Interpreting the output

1. **Test Type**: We performed a "2-sample test for equality of proportions without continuity correction". This means we compared two independent proportions (proportions of men and women who purchased the product in our scenario). The 'without continuity correction' part indicates that we didn't apply any correction for continuity, suitable for large sample sizes.

2. **Data**: "`x out of n`" refers to our input data structure, where `x` represents the number of successes (men and women who purchased the product) and `n` represents the total number of observations (total men and women in our sample).

3. **X-squared**: The test statistic value is `5.9138`. In a two-proportion Z-test like ours, this statistic follows a `chi-square` distribution with `one` degree of freedom under the null hypothesis.

4. **Degrees of Freedom (df)**: It's 1 in our case. For a two-sample test of proportions, the degrees of freedom is typically n-1, where n is the number of groups (2 in our case), but since 2-1 equals 1, our df is 1.

5. **p-value**: Our p-value is `0.01502`. This measures the strength of the evidence against the null hypothesis. As it's less than the typical alpha level of `0.05`, we reject the null hypothesis, indicating a statistically significant difference between the proportions of men and women purchasing the product.

6. **Alternative Hypothesis**: "two.sided" - This indicates that our test was two-tailed, meaning *we were testing for any difference in proportions, not specifically if one proportion was greater or lesser than the other.*

7. **95 Percent Confidence Interval**: This interval, ranging `from -0.1805 to -0.0195`, provides a range of plausible values for the difference in proportions `(p1 - p2)`. Since this *interval doesn't include 0*, it suggests that the true difference in proportions is likely non-zero, reinforcing our conclusion from the p-value.
8. **Sample Estimates**: These are the observed proportions in our study - `prop 1` is `0.5` (proportion of men who purchased the product) and `prop 2` is `0.6` (proportion of women who purchased the product).

Conclusion: There is a statistically significant difference in the purchasing behavior between men and women for the product in our study, with the proportion of women purchasing the product being higher than that of men.

Another Demonstration using simulated data:

We can perform a one-sided test using the `prop.test` function in R by changing the `alternative` parameter from `"two.sided"` to either `"greater"` or `"less"`. This choice depends on the direction of the alternative hypothesis we wish to test.

In our scenario, if we aim to test whether the proportion of men who purchased the product is greater than the proportion of women, we would use `alternative = "greater"`. Conversely, if our hypothesis is that the proportion of men is less, we would use `alternative = "less"`.

Here's how we can modify our code for a one-sided test:

```r
x <- c(125, 210) # number of men and women who purchased the product
n <- c(250, 350) # total number of men and women

# Perform the one-sided two-proportion z-test
# We will use "less" to test if the proportion of men is less than that of women
# Change to "greater" if testing for the opposite
test_result <- prop.test(x,
                         n,
                         alternative = "less",
                         correct = FALSE)
# Output the results
print(test_result)
```

```
	2-sample test for equality of proportions without continuity correction

data:  x out of n
X-squared = 5.9138, df = 1, p-value = 0.007511
alternative hypothesis: less
95 percent confidence interval:
```

```
 -1.0000000 -0.0324665
sample estimates:
prop 1 prop 2
   0.5    0.6
```

In this code, we use `alternative = "less"` for a one-sided test where our hypothesis is that the proportion for the first group (men) is less than that for the second group (women). We need to choose the appropriate direction ("greater" or "less") based on our specific research hypothesis.

Interpreting the output

1. **X-squared**: The test statistic value is `5.9138`. This value is derived from the comparison of the two proportions.
2. **P-value**: The p-value is `0.007511`. In a one-sided test, this value indicates the probability of observing a test statistic as extreme as, or more extreme than, the one observed, under the null hypothesis. Since it's less than `0.05`, it suggests significant evidence against the null hypothesis in favor of the alternative.
3. **Alternative Hypothesis**: "`less`" - This specifies that the test is one-sided, testing whether the first proportion (`prop 1`) is less than the second proportion (`prop 2`).
4. **95 Percent Confidence Interval**: Ranges from `-1.000 to -0.0325`. This interval indicates where the true difference in proportions (`prop 1 - prop 2`) likely lies. Since the interval does not include 0 and is entirely negative, it suggests that `prop 1` is likely less than `prop 2`.
5. **Sample Estimates**: Shows the estimated proportions for each group - 0.5 (`prop 1`) and 0.6 (`prop 2`).

Conclusion: The test results suggest that there is a significant difference between the two groups, with the first group having a lower proportion compared to the second group, supporting the alternative hypothesis that the proportion of men purchasing the product is less than that of women.

Running Independent Samples Z-Test in R (2) – using `mtcars`

Objective: We aim to compare the proportion of cars with automatic transmission (indicated by `am` = 1) to those with V-shaped engines (indicated by `vs` = 1) in the `mtcars` dataset. Our goal is to determine if there is a significant difference in the proportions of these two categories of cars.

```r
# Loading the mtcars dataset
# Loading the mtcars dataset
data(mtcars)

# Defining the samples
# sample1 represents cars with automatic transmission (am = 1)
# sample2 represents cars with V-shaped engines (vs = 1)
sample1 <- mtcars$am
sample2 <- mtcars$vs

# Calculating the number of successes in each sample
# Success for sample1 is defined as having an automatic transmission (am = 1)
# Success for sample2 is defined as having a V-shaped engine (vs = 1)
successes_sample1 <- sum(sample1 == 1)
successes_sample2 <- sum(sample2 == 1)

# Calculating the sample sizes
# The sample size for each group is the total number of observations in that group
size_sample1 <- length(sample1)
size_sample2 <- length(sample2)

# Performing the Z-test for proportions
# The test compares the proportion of cars with automatic transmission to those with V-sha
prop.test(x = c(successes_sample1, successes_sample2),
          n = c(size_sample1, size_sample2),
          alternative = "two.sided")
```

```
	2-sample test for equality of proportions with continuity correction

data:  c(successes_sample1, successes_sample2) out of c(size_sample1, size_sample2)
X-squared = 0, df = 1, p-value = 1
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.3043652  0.2418652
sample estimates:
 prop 1  prop 2 
0.40625 0.43750 
```

Interpreting the output

1. **Test Type**: "2-sample test for equality of proportions with continuity correction" - This test compares two independent proportions (in this case, cars with automatic transmis-

sion vs. cars with V-shaped engines in the `mtcars` dataset). The continuity correction is applied, which is typically used for small sample sizes to improve the approximation of the binomial distribution to the normal distribution.

2. **X-squared**: The test statistic value is 0. This value is used to determine the p-value. An X-squared of 0 usually indicates no observed difference between the proportions.
3. **p-value**: With a p-value of 1, there's no statistical evidence to suggest a significant difference between the two proportions.
4. **Alternative Hypothesis**: "two.sided" - The test was conducted to determine if there is any difference (either way) between the two proportions.
5. **95 Percent Confidence Interval**: Ranges from -0.3044 to 0.2418. Since this interval includes 0, it suggests that the difference in proportions might be zero, supporting the lack of statistical significance.
6. **Sample Estimates**: The estimated proportions are 0.40625 for the first group (automatic transmissions) and 0.43750 for the second group (V-shaped engines).

Conclusion: The test suggests no significant difference between the proportion of cars with automatic transmissions and those with V-shaped engines in the mtcars dataset. [8]

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